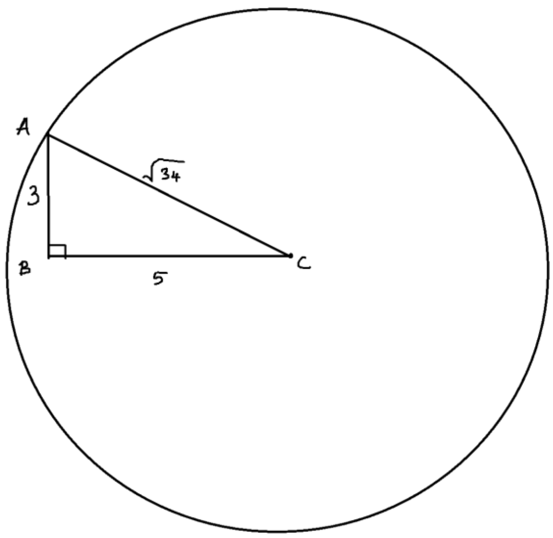
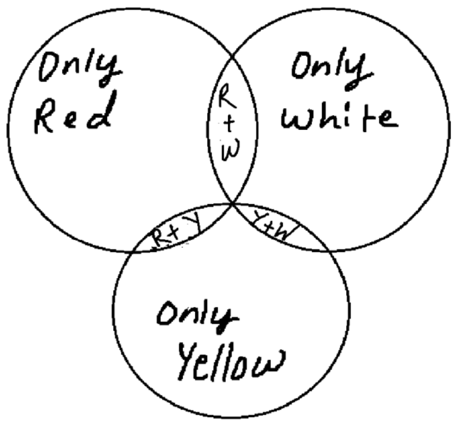
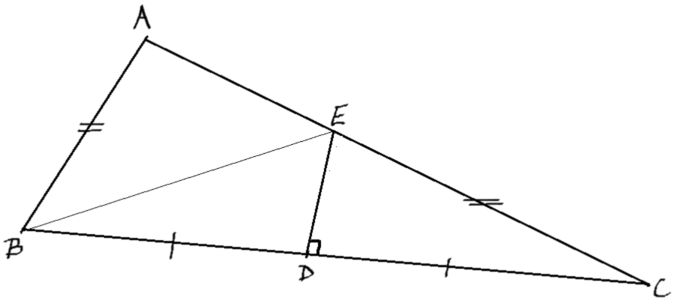
Ganit Prabhutva – Level II – 8th – Solutions

1	$Let\ 4048 = x\ ; 4047 = x - 1\ ; 4049 = x + 1$ $Thus\ \frac{(4048^2 + 1)}{(4049^2 + 4047^2)} = \frac{x^2 + 1}{(x + 1)^2 + (x - 1)^2}$ $= \frac{x^2 + 1}{x^2 + 2x + 1 + x^2 - 2x + 1}$ $= \frac{x^2 + 1}{2x^2 + 2}$ $= \frac{x^2 + 1}{2(x^2 + 1)}$ $= \frac{1}{2}$
2	$5\frac{1}{x} = \frac{5x + 1}{x}\ and\ y\frac{3}{4} = \frac{4y + 3}{4}$ $Thus\ 5\frac{1}{x} * y\frac{3}{4} = \left(\frac{5x + 1}{x}\right)\left(\frac{4y + 3}{4}\right) = 20$ $i.e. (5x + 1)(4y + 3) = 20 \times 4 \times x$ $Thus\ (5x + 1)(4y + 3) = 20 \times 4x$ $OR\ (5x + 1)(4y + 3) = 40 \times 2x$ $OR\ (5x + 1)(4y + 3) = 16 \times 5x$ $But\ x\ \&\ y\ are\ positive\ intergers.$ $Hence, (5x + 1)(4y + 3) = 16 \times 5x$ $i.e. (5x + 1) = 16\ \&\ (4y + 3) = 5x$ $x = 3\ \&\ y = 3$
3	$\sqrt{m\sqrt{m\sqrt{m}}} = 128$ $Squaring\ both\ sides$ $m * \sqrt{m * \sqrt{m}} = 128^2 = (2^7)^2 = 2^{14}$ $Again\ squaring\ both\ sides$ $m^2 m\sqrt{m} = (2^{14})^2 = 2^{28}$ $Squaring\ again$ $m^4 m^2 m = (2^{28})^2 = 2^{56}$ $m^{4+2+1} = 2^{56}$ $m^7 = 2^{56}$ $m^7 = (2^8)^7$ $m = 2^8 = 256$

4	$A = \pi r^2$ $34\pi = \pi r^2$ $r = \sqrt{34}$ As $\sqrt{34}$ is an irrational number & cannot be measured on scale. $3^2 + 5^2 = 34$ Hence, $\sqrt{34}$ is the hypotenuse of right-angled triangle of sides 3 cm & 5 cm. Construct $\Delta\, ABC$ as shown Take distance AC in compass & draw a circle. This is the circle with radius of $\sqrt{34}$ & area 34π. 
5	$Total\ Area = A(Red) + A(White) + A(Yellow) - \{A(R\&W) + A(Y\&W) + A(R\&Y)\}$ $\ldots 100\% = 70\% + 40\% + 30\% - \{A(R\&W) + A(Y\&W) + A(R\&Y)\}$ $\ldots \{A(R\&W) + A(Y\&W) + A(R\&Y)\} = 40\%$ $Total\ Area = A(Only\ Red) + A(Only\ White) + A(Only\ Yellow) + \{A(R\&W) + A(Y\&W) + A(R\&Y)\}$ $\ldots 100\% = A(Only\ Red) + A(Only\ White) + A(Only\ Yellow) + 40\%$ $\ldots A(Only\ Red) + A(Only\ White) + A(Only\ Yellow) = 100\% - 40\% = 60\%$ 

6	$E(1)+E(2)+E(3)+\cdots+E(100)$ $=\{E(1)+E(2)+\cdots+E(10)\}+\{E(11)+E(12)+\cdots E(20)\}+\cdots\{E(91)+E(92)+\cdots+E(100)\}$ $=\{0+2+0+4+0+6+0+8+0+0\}*10+\{2*10+4*10+6*10+8*10\}$ $=\{2+4+6+8\}*20$ $=400$
7	$Let\ the\ original\ number\ be\ x$ $New\ number\ is\ x+10$ $\%\ increase=\frac{New-Original}{New}*100$ $\%\ increase=\frac{x+10-x}{x}*100$ $\%\ increase=\frac{1000}{x}$ $72=(x+10)+\left[\frac{1000}{x}\%\ of\ (x+10)\right]$ $72=(x+10)+\left[\frac{10}{x}*(x+10)\right]$ $72x=x(x+10)+10(x+10)$ $72x=x^2+10x+10x+100$ $x^2-52x+100=0$ $x^2-2x-50x+100=0$ $x(x-2)-50(x-2)=0$ $(x-50)(x-2)=0$ $x=50\ OR\ x=2$
8	$Let\ CP\ of\ each\ article\ be\ x$ $Thus\ number\ of\ articles\ bought=\frac{600}{x}$ $SP=x+5$ $Total\ Sale=SP*No.of\ articles\ sold$ $\ldots Total\ Sale=(x+5)(\frac{600}{x}-10)$ $\ldots Total\ Sale=\frac{(x+5)(600-10x)}{x}$ $\ldots Total\ Sale=\frac{-10x^2+550x+3000}{x}$ $\ldots Total\ Profit=Total\ Sale-Total\ CP$ $\ldots Total\ Profit=\frac{-10x^2+550x+3000}{x}-600$ $\ldots Total\ Profit=\frac{-10x^2+550x+3000-600x}{x}$ $\ldots Total\ Profit=\frac{-10x^2-50x+3000}{x}$ $But, Total\ Profit=CP\ of\ 15\ articles$

	$\ldots Total\ Profit = \dfrac{-10x^2 - 50x + 3000}{x} = 15x$ $\ldots -10x^2 - 50x + 3000 = 15x^2$ $\ldots 25x^2 + 50x - 3000 = 0$ $\ldots x^2 + 2x - 120 = 0$ $\ldots x^2 + 12x - 10x - 120 = 0$ $\ldots x(x + 12) - 10(x + 10) = 0$ $\ldots x = -12\ OR\ x = 10$ $But\ x \neq -12$ $Thus\ CP\ of\ each\ article\ is\ Rs. 10$
9	$a = \frac{1}{2}(\sqrt{3} + 1)$ $\ldots 2a = (\sqrt{3} + 1)$ $\ldots 2a - 1 = \sqrt{3}$ $\ldots (2a - 1)^2 = 3$ $\ldots 4a^2 - 4a - 2 = 0$ ….Eq. (1) $\ldots 2a^2 - 2a = 1$ ….Eq. (2) We have to find $4a^3 + 2a^2 - 8a + 7$ $\ldots 4a^3 - 4a^2 + 6a^2 - 2a - 6a + 7$ $\ldots 4a^3 - 4a^2 - 2a + 6a^2 - 6a + 7$ $\ldots a(4a^2 - 4a - 2) + 3(2a^2 - 2a) + 7$ $\ldots a(0) + 3(1) + 7$ $\ldots 10$
10	 $Draw\ seg\ BE$ $\Delta BED \cong \Delta CED\ using\ SAS\ test\ of\ congruency$ $\therefore seg\ BE = seg\ CE\ using\ corresponding\ sides\ of\ congruent\ triangles$ $\therefore \angle EBD \cong \angle ECD = x$ $\angle ECD\ is\ same\ as\ \angle ACB. Hence, \angle ACB = x$ $\angle AEB is\ exterior\ angle\ of\ \Delta BEC$ $\therefore \angle AEB = 2x$ $Since\ seg\ BE = seg\ CE\ \&\ seg\ AB = Seg\ EC. \therefore seg\ BE \cong segAB$ $\therefore \Delta AEB\ is\ isosceles\ \&\ \angle BAC \cong \angle AEB = 2x$ $In\ \Delta ABC, \angle ABC + \angle BAC + \angle ACB = 180°$ $\therefore 57° + 2x + x = 180°$ $\therefore 3x = 180 - 57 = 123$ $\therefore x = 41°$

11	Let a,b,c be the time required by taps A, B & C respectively to fill the entire tank. $\frac{1}{a}\times t+\frac{1}{b}\times 10+\frac{1}{c}\times(t-12)=1$ ….Eq.(1) $\frac{1}{a}+\frac{1}{b}=\frac{1}{t}$ …..Eq.(2) As each tap fills equal volume of the tank, it means each tap fills $1/3^{rd}$ volume. $\frac{t}{a}=\frac{10}{b}=\frac{t-12}{c}=\frac{1}{3}$ …..Eq.(3) $b=30min$ Substituting b in Eq.(2), $\frac{1}{a}+\frac{1}{30}=\frac{1}{t}$ $\frac{t}{a}=1-\frac{t}{30}$ $But\,\frac{t}{a}=\frac{1}{3}$ $1-\frac{t}{30}=\frac{1}{3}$ $t=20\;min$ Using Eq.(3), $\frac{t-12}{c}=\frac{1}{3}$ $c=24\;min$
12	Let $x=a*h\;and\;y=b*h$ where h is HCF of $x\;and\;y$ Hence, a & b are co-prime numbers. $x+y=1050$ $\therefore ah+bh=1050$ $\therefore 1050=h(a+b)$ $\therefore 2\times3\times5\times5\times7=h(a+b)$ Since h must be maximum, $a+b$ should be minimum. $If\;a+b=2, a=1\;\&\;b=1$ $i\cdot e\cdot x=y$ But x & y are distinct. Hence, $a+b\neq 2$. $If\;a+b=3$ $\therefore a=1\;and\;b=2$ $\therefore h=2\times5\times5\times7=350$ $\therefore x=1\times350$ $\therefore y=2\times350$ $h=350\;;x=350\;;y=700$

13

$Let\ M\ be\ the\ point\ of\ meet$

$Let\ S_c\ \&\ S_t\ be\ the\ speeds\ of\ car\ \&\ truck\ respectively$

$Car\ travels\ P\ to\ Q\ from\ 9{:}\,00am\ to\ 1{:}\,00pm\ i.e\ in\ 4\ hrs$

$PQ = S_c * T = 90 * 4 = 360\ km$

$QM = \frac{1}{4} * 360 = 90\ km$

$PM = 360 - 90 = 270\ km$

$Thus, time\ taken\ by\ car\ to\ reach\ pt.M = \frac{QM}{S_c} = \frac{270}{90} = 3\ hrs$

$Thus\ time\ of\ meet\ is\ 3\ hrs\ after\ 9{:}\,00am\ i.e.at\ 12{:}\,00pm$

$Truck\ covers\ distance\ QM\ from\ 10{:}\,00am\ till\ 12{:}\,00pm\ i.e.in\ 2\ hrs$

$S_t = \frac{QM}{Time} = \frac{90}{2} = 45kmph$

$Thus, time\ taken\ by\ truck\ to\ cover\ QP = \frac{QP}{S_t} = \frac{360}{45} = 8\ hrs$

14

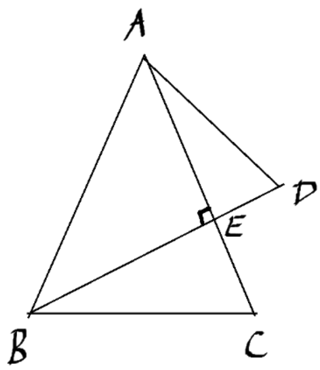
$Let\ \angle BAC = x\ \&\ \angle ABD = y$

$As\ \Delta ABC\ is\ isosceles, \angle ABC = \angle ACB$

$= \frac{180 - x}{2}$

$As\ \Delta BAD\ is\ isosceles, \angle BAD = \angle ADB$

$= \frac{180 - y}{2}$



$In\ \Delta AEB\ , \angle AEB + \angle ABD + \angle BAC = 180$

$\ldots 90 + y + x = 180$

$\ldots x + y = 90$

$\angle ACB + \angle ADB = \frac{180 - x}{2} + \frac{180 - y}{2}$

$\ldots \angle ACB + \angle ADB = 180 - \frac{x + y}{2}$

$\ldots \angle ACB + \angle ADB = 180 - \frac{90}{2}$

$\ldots \angle ACB + \angle ADB = 135$

15	$Avg\ marks\ of\ 10\ students = \frac{Total\ marks\ of\ 10\ students}{No.of\ students}$ $\dots Total\ marks\ of 10\ students = 60 * 10 = 600$ $Avg\ marks\ of\ bottom\ 5\ students = 60 - 5 = 55$ $\dots Avg\ marks\ of\ bottom\ 5\ students = \frac{Total\ marks\ of\ bottom\ 5\ students}{5}$ $\dots Total\ marks\ of\ bottom\ 5\ students = 55 * 5 = 275$ $\dots Total\ marks\ of\ top\ 5\ students$ $= Total\ marks\ of\ 10\ students$ $- Total\ marks\ of\ bottom\ 5\ students$ $\dots Total\ marks\ of\ top\ 5\ students = 600 - 275 = 325$ $\dots Let\ the\ distinct\ scores\ of\ top\ 5\ students\ be\ a, b, c, d, e$ $\dots a + b + c + d + e = 325$ $\dots If\ a\ must\ be\ maximum, b, c, d, e\ should\ be\ as\ low\ as\ possible$ $Also, 6th\ ranker'score < e$ and $the\ average\ of\ bottom\ 5\ rankers\ is\ 55$ $\dots As\ e\ is\ as\ low\ as\ possible, it\ means\ 6th\ ranker's\ score\ is\ also\ as\ low\ as$ $possible\ \&\ avg\ is\ 55$ $In\ bottom\ 5, if\ one\ scores\ less\ than\ 55, other\ must\ score\ more\ than$ $55\ to\ maintain\ the\ average\ at\ 55$ $\dots All\ the\ bottom\ 5\ should\ score\ 55\ marks, so\ that\ 6th\ ranker's\ score\ is\ low$ $\dots e = 56, d = 57, c = 58, b = 59$ $\dots a + b + c + d + e = 325$ $\dots a = 325 - (56 + 57 + 58 + 59)$ $\dots a = 95$