

Ganit Prabhutwa – Level II (2024-2025) – 5th - Model Solution

1. A two-digit number is 7 times the sum of its digits. Find all such numbers. If the digits of each such number are interchanged, how many times will the number be of sum of its digits?

Two-digit multiples of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98

$$21 = 7 \times 3 = 7 \times (2 + 1) = 7 \times \text{sum of the digits}$$

$$42 = 7 \times 6 = 7 \times (4 + 2) = 7 \times \text{sum of the digits}$$

$$63 = 7 \times 9 = 7 \times (6 + 3) = 7 \times \text{sum of the digits}$$

$$84 = 7 \times 12 = 7 \times (8 + 4) = 7 \times \text{sum of the digits}$$

\therefore each of the numbers 21, 42, 63, 84 is 7 times the sum of its digits.

By interchanging the digits of these numbers, we get the numbers 12, 24, 36, 48]

$$12 = 4 \times 3 = 4 \times (1 + 2) = 4 \times \text{sum of the digits}$$

$$24 = 4 \times 6 = 4 \times (2 + 4) = 4 \times \text{sum of the digits}$$

$$36 = 4 \times 9 = 4 \times (3 + 6) = 4 \times \text{sum of the digits}$$

$$48 = 4 \times 12 = 4 \times (4 + 8) = 4 \times \text{sum of the digits}$$

\therefore Each number of them is 4 times the sum of its digits

2. 3 fractions, having equal numerators, have their denominators 2, 4 and 8. If the sum of the fractions is $\frac{21}{8}$, what is the numerator of the said fractions?

Let the fractions be $\frac{x}{2}$, $\frac{x}{4}$, $\frac{x}{8}$

L. C. M. of the denominators = 8

$$\therefore \frac{x}{2} + \frac{x}{4} + \frac{x}{8} = 4 \times \frac{x}{8} + 2 \times \frac{x}{8} + 1 \times \frac{x}{8}$$

$$= (4 + 2 + 1) \times \frac{x}{8} = 7 \times \frac{x}{8}$$

$$\text{Sum of the fractions} = \frac{21}{8} \text{ (Given)}$$

$$\therefore 7 \times \frac{x}{8} = \frac{21}{8}$$

$$\therefore \frac{x}{8} = \frac{3}{8} \quad \therefore 4 \times \frac{x}{8} = \frac{12}{8} = \frac{3}{2}, \quad 2 \times \frac{x}{8} = \frac{6}{8} = \frac{3}{4} \text{ and } 1 \times \frac{x}{8} = \frac{3}{8}$$

So, the fractions are $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$

3. If 1 student from class A is transferred to class B, then the two classes will have the same number of students. But if 1 student from class B is transferred to class C, then class C will have 2 more students than class B. What is the difference between the number of students in class A and C initially?

$$A - 1 = B + 1 \text{ So, } A - B = 2$$

That means class A has 2 students more than class B.

$$B - 1 = C + 1 - 2 \text{ So, } B - C = 0$$

That means class B has 0 students more than class C

Class A has 2 students more than class B and Class B has 0 students more than class C

\therefore class A has 2 students more than class C \therefore The difference is 2

4. Amit bought an article for Rs 1000. He sold it to Bipin with 10% profit. Bipin sold it to Chetan with 10% profit on his cost price. Find the price at which Chetan bought the article. Chetan sold it to Dinesh such that the selling price was ten times the loss, find the selling price of Chetan and loss percent.

C. P. for Amit = 1000. He sold it with 10% profit. Profit = $1000 \times 10/100 = 100$.

$$\therefore \text{S.P.} = 1000 + 100 = 1100$$

C. P. for Bipin = 1100. His profit = $1100 \times 10/100 = 110$.

$$\therefore \text{S.P.} = 1100 + 110 = 1210$$

C. P. for Chetan = 1210. He sold it to Dinesh with the selling price 10 times the loss.

$$\text{S. P.} + \text{loss} = \text{C. P.} \therefore 10(\text{loss}) + \text{loss} = \text{C. P.}$$

$$\therefore 11(\text{loss}) = 1210. \text{ So, } \text{loss} = 1210/11 = 110$$

$$\therefore \text{selling price for Chetan} = 1210 - 110 = 1100 \text{ Rs.}$$

$$\text{His loss percent} = 110/1210 \times 100\% = 100/11 \%$$

5. There are 25 rows of seats in an auditorium. The first row has 20 seats. The second row has 22 seats. The third row has 24 seats and so on. How many seats are there in the last row? Also, find the total number of seats in the auditorium.

The sequence of seats will be 20, 22, 24, --- \therefore The common difference is 2

$$\therefore t(1) = 20 + 0, t(2) = 20 + 2, t(3) = 20 + 4, t(4) = 20 + 6$$

$$\therefore t(1) = 20 + 2 \times 0; t(2) = 20 + 2 \times 1, t(3) = 20 + 2 \times 2, t(4) = 20 + 2 \times 3 \text{ and so on}$$

$$\therefore t(25) = 20 + 2 \times 24 = 20 + 48 = 68$$

$$\text{Sum} = 20 + 22 + 24 + \dots + 68 = 25 \times \text{the middle term}$$

$$= 25 \times \text{thirteenth term} = 25 \times 44 = 1100$$

6. There were 1580 people on a ship. 60% of them were adults. When the ship halted at a harbor, some adults alighted. Then the percentage of the adults on the ship became 20. How many children were there on the ship? How many adults alighted from the ship?

60% of 1580 = $1580 \times 60/100 = 948$. \therefore there were 948 adults on the ship

$1580 - 948 = 632$. \therefore there were 632 children on the ship.

When some adults alighted, the percentage of adults became 20.

\therefore the percentage of children became 80

But the number of children is 632

$\therefore 632 \times 100 \div 80 = 790$ was the total number of persons on the ship

Out of them 632 were children $\therefore 790 - 632 = 158$ adults remained on the ship.

$\therefore 948 - 158 = 790$ adults alighted from the ship.

7. The rectangular coloured paper has a length of 11 cm and width of 5 cm. How many maximum pieces of 3 cm \times 2 cm rectangles can be cut out from that coloured paper with minimum wastage? Explain with a proper sketch.

8. Using the digits 5, 6, and 7 only, each at least once, how many four-digit numbers can be formed which will be divisible by 11? What will be the difference between the greatest and the smallest of such numbers?

To get four-digit numbers, one of the digits 5, 6, 7 is to be taken twice

As the number is to be divisible by 11, sum of the repeated digits should be equal to the sum of remaining digits. \therefore the possible digits are 5, 6, 6, 7

\therefore the possible numbers will be 5676, 6567, 7656, and 6765

The greatest number = 7656 and the smallest number = 5676

The difference = $7656 - 5676 = 1980$

9. In a competitive examination of 100 marks each question carried 5 marks. 2 marks were deducted for every wrong answer and 1 mark was deducted for every non attempted question. Rohan answered 14 questions correctly. But he obtained only 60 marks. How many questions were wrong and how many questions he did not attempt?

Total marks 100 and each question carried 5 marks, \therefore there were 20 questions.
Rohan's 14 answers were correct. \therefore he would get 70 marks for correct answers
But he got 10 marks less. \therefore he did not attempt some questions and/or got some answers wrong

If his 6 questions were wrong, he would have got $70 - 12 = 58$ marks.

If 5 questions were wrong and 1 question was not attempted, then he would have got $70 - 10 - 1 = 59$ marks

If 4 questions were wrong and 2 questions were not attempted, then he would have got $70 - 8 - 2 = 60$ marks

\therefore Rohan's 4 answers were wrong and he did not attempt 2 questions.

10. Six friends shared a box of mangoes. The first one took $\frac{1}{6}$ th of them. The second one took $\frac{1}{5}$ th of the remaining. The third one took $\frac{1}{4}$ th of the remaining. The fourth one took $\frac{1}{3}$ rd of the remaining and the last two friends got 8 mangoes each. Find the number of mangoes in the box find the share of each of them.

$1 - \frac{1}{6} = \frac{5}{6}$ \therefore when the first one took $\frac{1}{6}$, $\frac{5}{6}$ remained.

Then, $\frac{1}{5}$ of $\frac{5}{6} = \frac{1}{6}$ were taken by the second person.

Again, $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ remained.

Then, the third one took $\frac{1}{4}$ of $\frac{4}{6} = \frac{1}{6}$. Again, $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$ remained.

The fourth one took $\frac{1}{4}$ of $\frac{3}{6} = \frac{1}{6}$

$\therefore \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$ remained, which were taken by the last 2 persons.

But both of them got 8 mangoes each. $\therefore \frac{1}{6}$ part = 8

\therefore Each one of them got 8 mangoes and there were 48 mangoes in the box.

11. A printer used total 600 digits for numbering all pages of a book. How many pages are there in the book?

The number of digits used for the pages from 1 to 9 = $9 = 9$

The number of digits used for the pages from 10 to 99 = $90 \times 2 = 180$

The digits used for the pages were from 100 to 199 = $100 \times 3 = 300$

$\therefore 300 + 180 + 9 = 489$ digits were used for 1 to 199 pages.

$\therefore 600 - 489 = 111$ digits remained and $111 \div 3 = 37$

$\therefore 111$ digits are used for 37 pages. \therefore In all there were $199 + 37 = 236$ pages

12. When $\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$ is divided by $\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)$ the result is equal to $10 \times A\%$. Then what is the value of A?

L.C.M. of the denominators of $\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$ is 60

$$\therefore \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right) = \left(\frac{30}{60} - \frac{15}{60} + \frac{12}{60} - \frac{10}{60}\right) = \frac{17}{60}$$

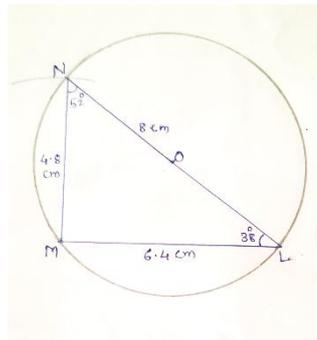
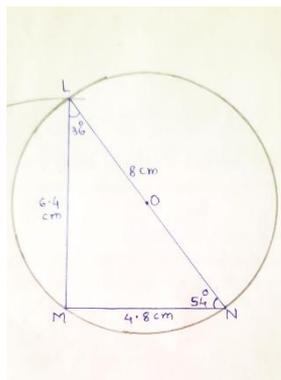
L.C.M. of the denominators of $\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)$ is 90

$$\therefore \left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right) = \left(\frac{36}{90} - \frac{50}{90} + \frac{54}{90} - \frac{35}{90}\right) = \frac{5}{90} = \frac{1}{18}$$

$$\therefore \frac{17}{60} \div \frac{1}{18} = \frac{17}{60} \times \frac{18}{1} = \frac{51}{10} = 10 \times A\%$$

$$\therefore \frac{51}{10} = 10 \times \frac{A}{100} \quad \therefore \frac{51}{10} = \frac{A}{10} \quad \therefore A = 51$$

13. Draw a $\triangle LMN$ in which $l(LM) = 6.4$ cm, $l(MN) = 4.8$ cm and $l(LN) = 8.0$ cm. Draw a circle whose diameter is side LN. Name the centre of the circle as O. Measure and write $\angle MLN$ and $\angle MNL$



Drawing triangle LMN with correct measurements

Taking the midpoint O of LN, such that the radius = 4 cm.

Drawing the circle on diameter LN

Measuring and writing measures of $\angle MLN \approx 37^\circ$ and $\angle MNL \approx 53^\circ$

14. A and B are three-digit odd numbers. H.C.F. of A and B is 55. A is a multiple of 9. Find all possible options for A and B which will satisfy all the above conditions.

H.C.F of two numbers = 55. \therefore both the numbers are divisible by 11 and 5

A is a multiple of 9. \therefore A is divisible by 5, 9 and 11.

$A = 5 \times 9 \times 11 = 495$ or any three-digit odd multiple of 495.

If $A = 495$, then B can be $55 \times 5 = 275$ or $55 \times 7 = 385$

Similarly B can be $55 \times 11, 55 \times 13, 55 \times 17$

The possible pairs of A and B satisfying the conditions are (495, 275), (495, 385), (495, 605), (495, 715), (495, 935)

15. ABCD is a square formed by joining 4 identical rectangles. The perimeter of each rectangle is 80 cm. What is the area of the square so formed? Explain with sketch showing arrangement of 4 rectangles.



Fig. 1

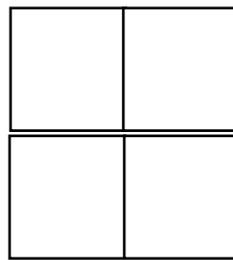


Fig.2

Fig. 1 shows 4 identical rectangles having length 32 cm and breadth 8 cm.

$$\therefore \text{Perimeter of each rectangle} = 2(32 + 8) = 80 \text{ cm.}$$

$$\therefore \text{Area of each rectangle is } 32 \times 8 = 256 \text{ cm}^2$$

$$\therefore \text{Area of the square formed} = 256 \times 4 = 1024 \text{ cm}^2$$

Figure 2 shows 4 identical squares. A square is a rectangle too.

$$\therefore \text{Side of each square} = 20 \text{ cm. Perimeter of each square} = 80 \text{ cm}$$

$$\therefore \text{Area of each square} = 20 \times 20 = 400 \text{ cm}^2$$

$$\therefore \text{Area of the big square formed} = 400 \times 4 = 1600 \text{ cm}^2$$

16. A and B were standing at one corner of the given symmetric figure. Both decided to walk along the side of the figure as shown in the figure. A was walking with speed 12 m/min while B was walking with speed 15 m/min. If they had started walking at 8 a.m., at what time they met each other and how far from each one was their original place?

